The Size of Message Set Needed for the Optimal Communication Policy

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Background

- Multi-agent coordination with communication.

> Main objective :
> To find the optimal *action policy* δ^A and *communication policy* δ^M

- We are interested in an approach based on autonomous learning.

- Definition of policies for agent i in our proposed methods

	Signal Learning (SL) [Kasai+ 08]	**Signal Learning with Messages (SLM)** [Kasai+ AAMAS09]
Action Policy	$\delta_i^A : \Omega_i \times \boldsymbol{M}_i^{receive} \rightarrow A_i$	
Communi-cation Policy	$\delta_i^M : \Omega_i \rightarrow M_i$	$\delta_i^M : \Omega_i \times \boldsymbol{M}_i^{receive} \rightarrow M_i$

A set of observations

A set of actions

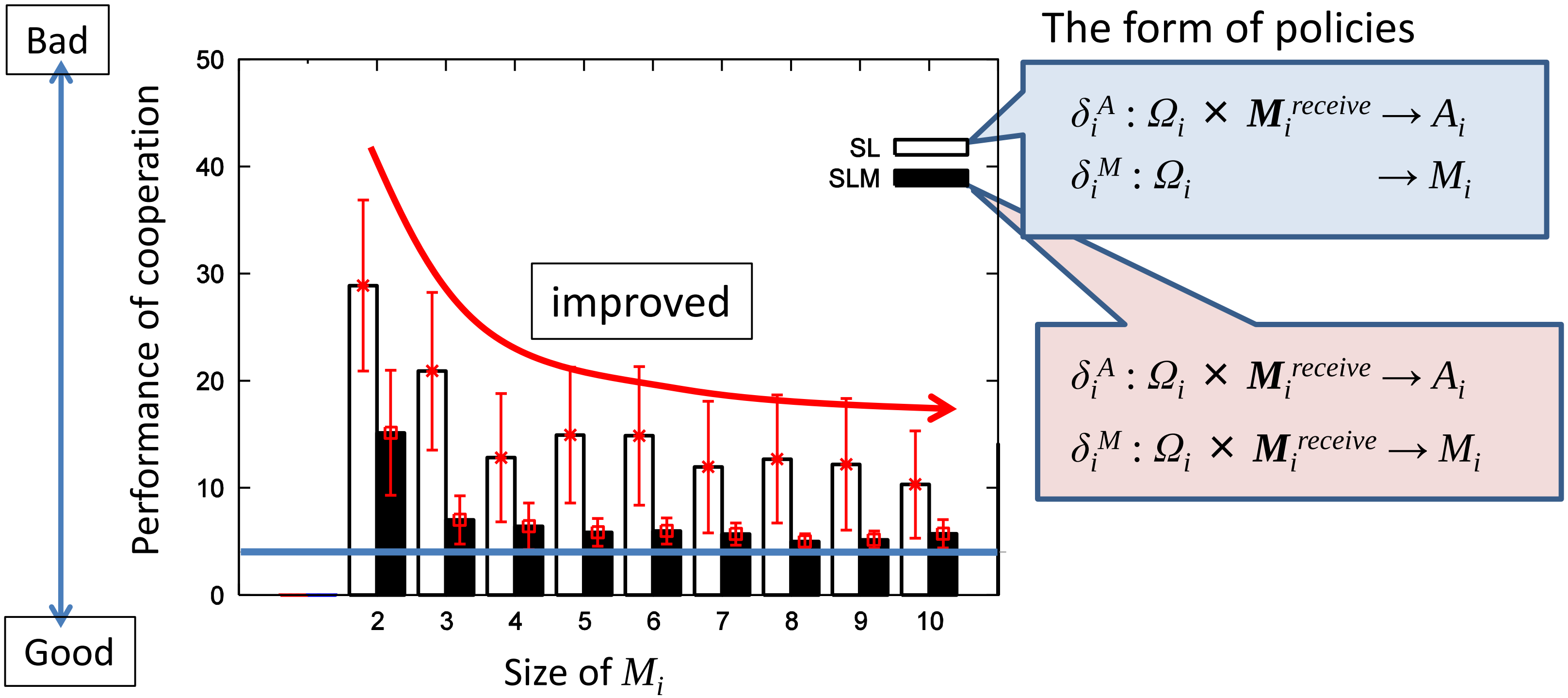
A set of received messages

A set of messages to send other agents

(SL and SLM are based on Multi-Agent Reinforcement Learning framework)

Motivation

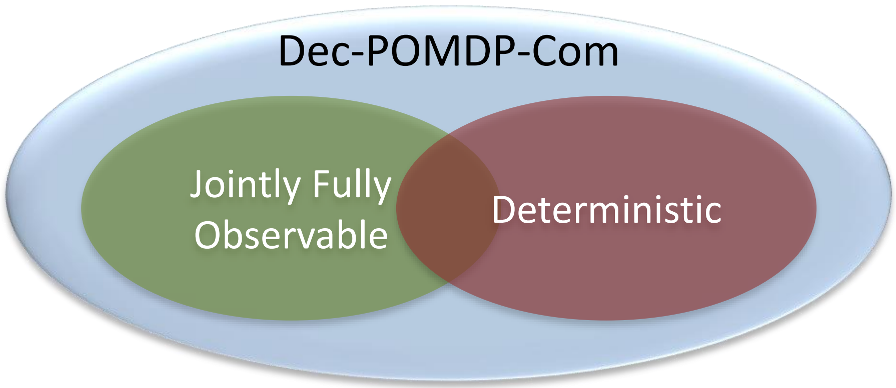
- Actual learning results of SL and SLM [Kasai+ AAMAS09]
  - The performance of cooperation when the size of M_i is increases.



- We have an interest about how much size of M_i for constructing the optimal policy ?

Scheme of talk

- We show *minimum required sizes* $|M_i|$ for achieving the optimal policy for
  - Signal Learning on *Jointly Fully Observable* *Dec-POMDP-Com*
  - Signal Learning with Messages on *Deterministic* *Dec-POMDP-Com*



Outline

- ☑ Background
- ☑ Scheme of talk
- ☐ Review : Dec-POMDP-Com [Goldman+ 04]
- ☐ Constrained model
  - Jointly Fully Observable Dec-POMDP-Com [Goldman+ 04]
  - Deterministic Dec-POMDP-Com (we define)
- ☐ Theoretical analysis
- ☐ Conclusion

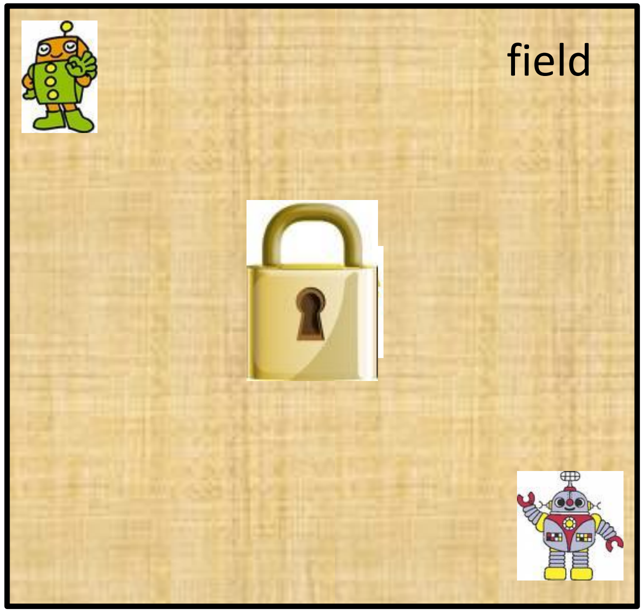
Dec-POMDP-Com [Goldman+ 04]

(Decentralized Partially Observable Markov Decision Process with Communication)

- A decentralized multi-agent system, where agents can communicate with each other and only observe the restricted information.

- Two agents get a treasure cooperatively.
- The treasure is locked.
- Both agents must reach the treasure at the same time to open the lock.

Example of model



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1step for agent i on Dec-POMDP-Com

1. Receive an observation O_i from the environment.
2. Send a message m_i to the other agents.
3. Perform an action a_i in the environment.

Repeat until both agent arrive at the treasure.

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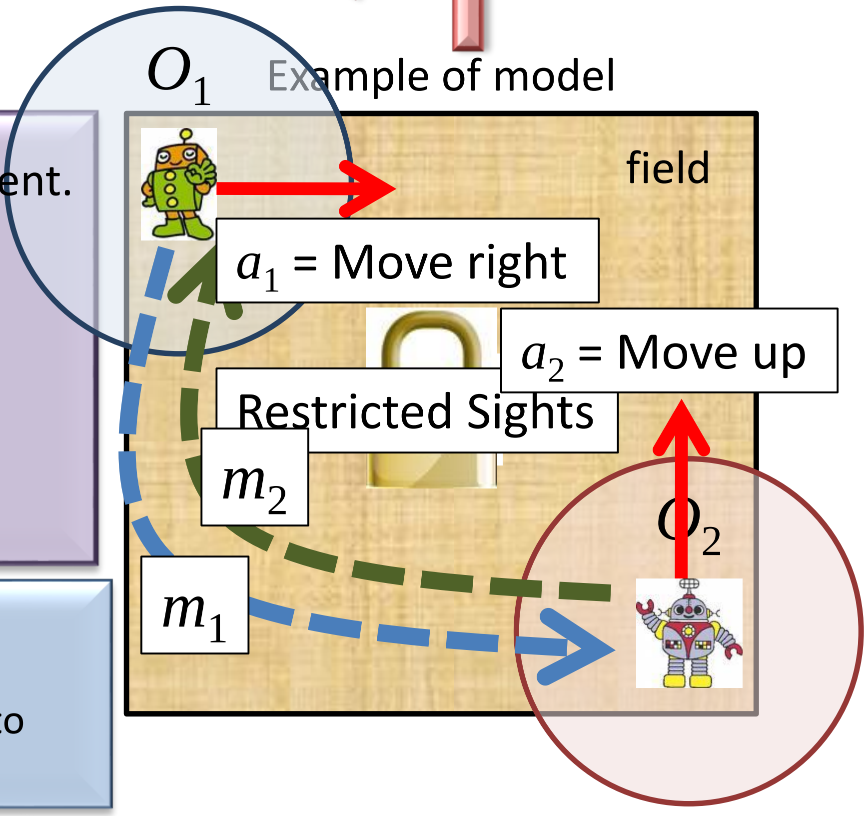

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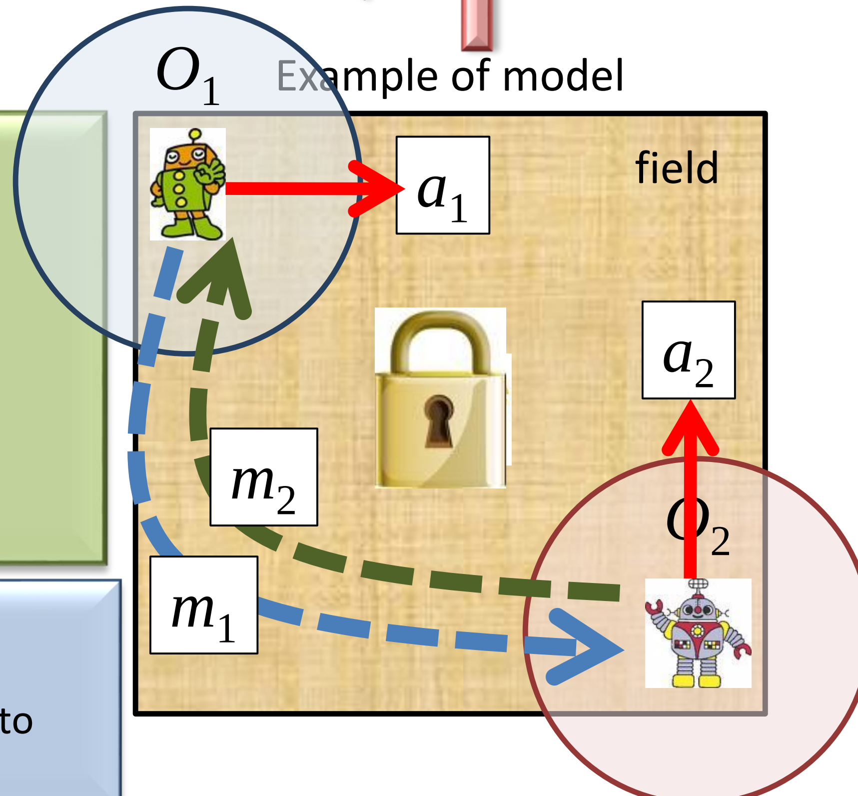
Formulation

- A set of agents' indices
- e.g., $I = \{1, 2\}$

Example of model



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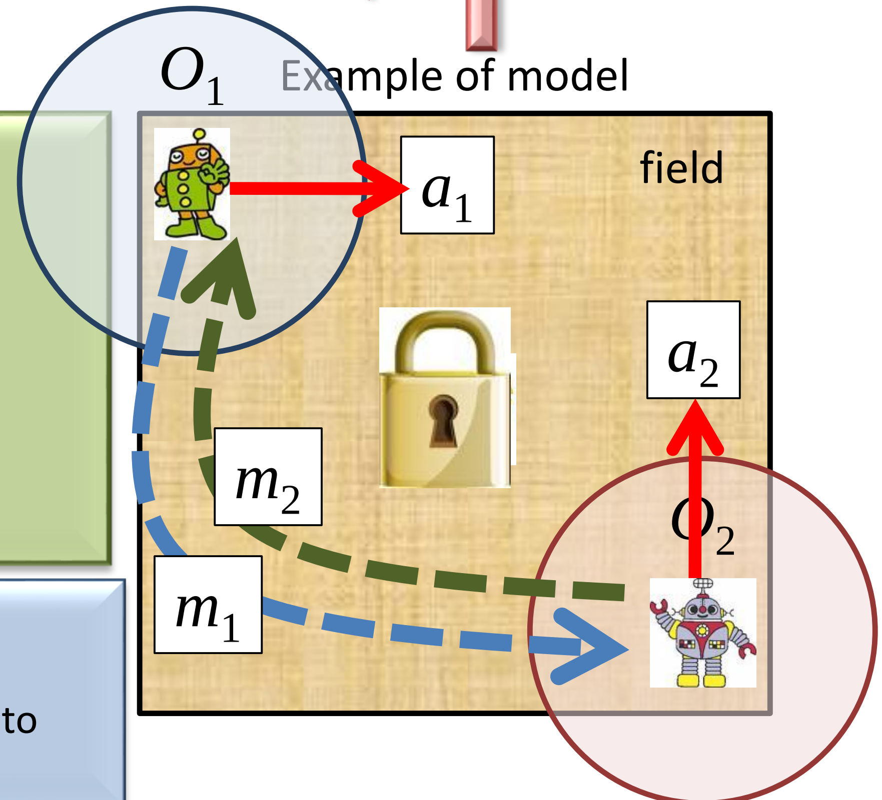
- A set of global states
- e.g., s = (position of agent 1,
  position of agent 2,
  position of treasure)
  , $s \in S$



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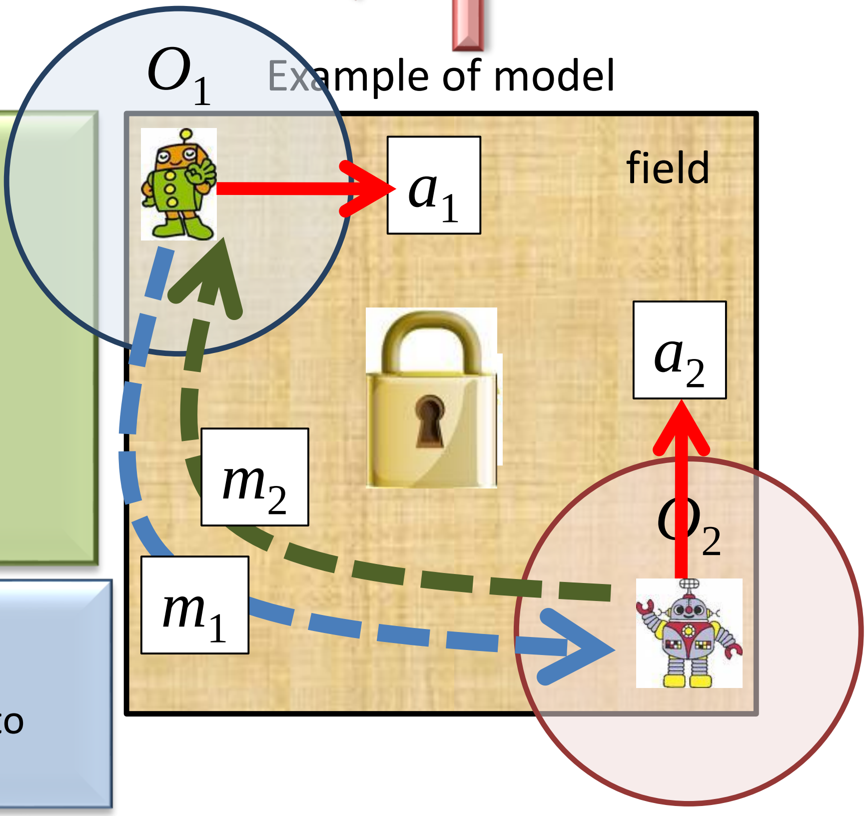
- $\boldsymbol{\Omega}$: a set of joint observations
- $\boldsymbol{\Omega} = \Omega_1 \times \Omega_2$,
  where Ω_i is a set of observations for agent i
- $\boldsymbol{A}$: a set of joint actions
- $\boldsymbol{A} = A_1 \times A_2$

Example of model



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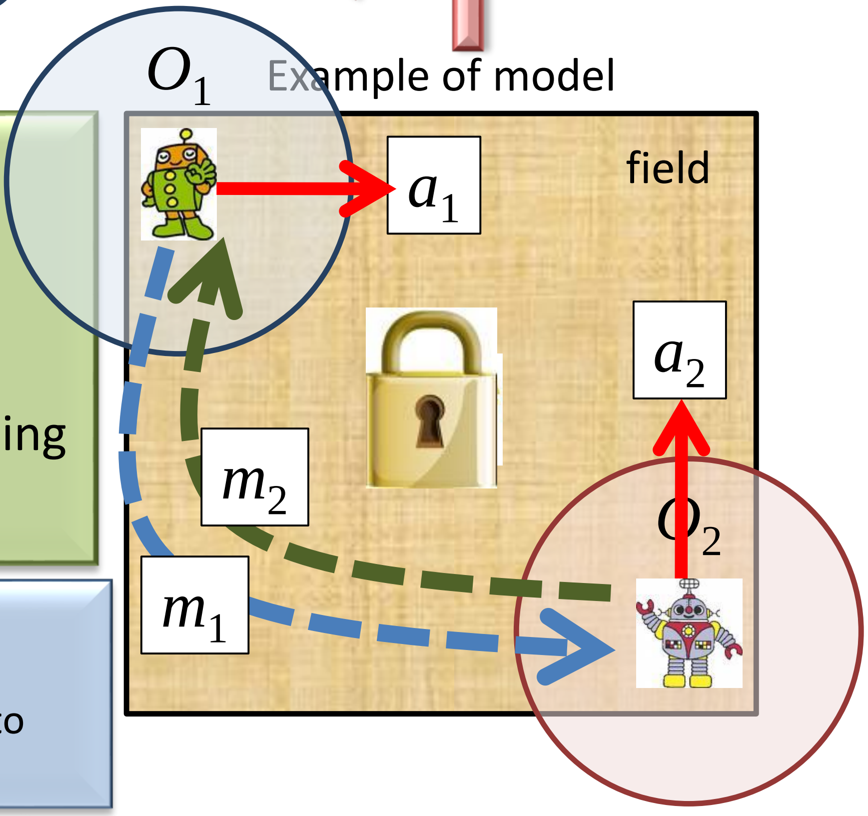
- $\boldsymbol{M}$: a set of joint messages
- $\boldsymbol{M} = M_1 \times M_2$
- $C : \boldsymbol{M} \rightarrow \mathfrak{R}$ is a cost function
- $C(\boldsymbol{m})$ represent the total cost of transmitting the messages sent by all agents.

Example of model



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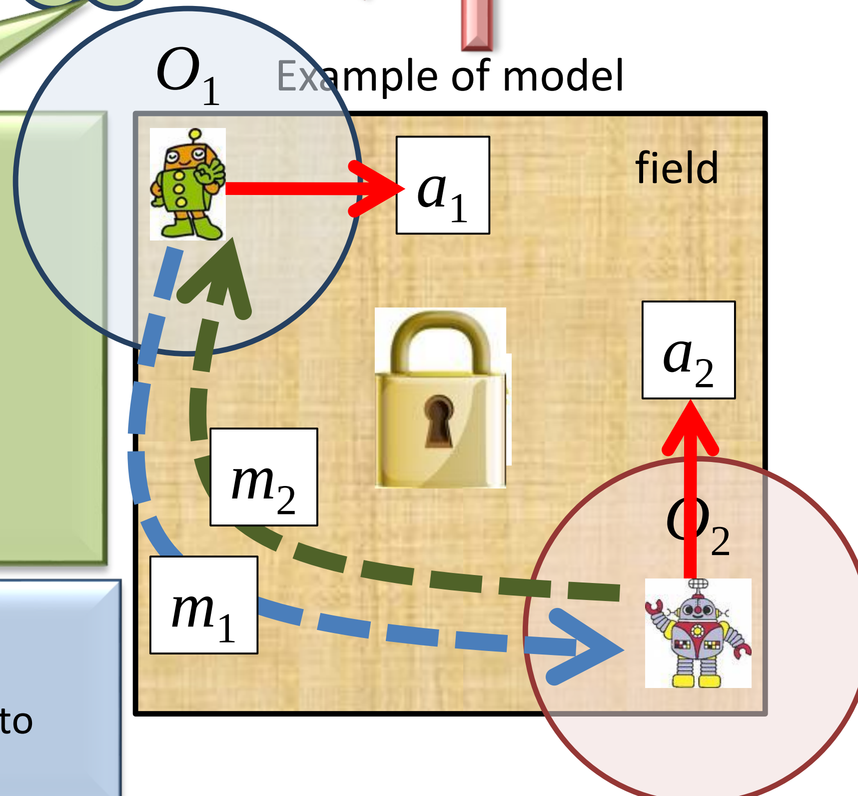
Formulation

- P : a transition probability function
- O : an observation probability function



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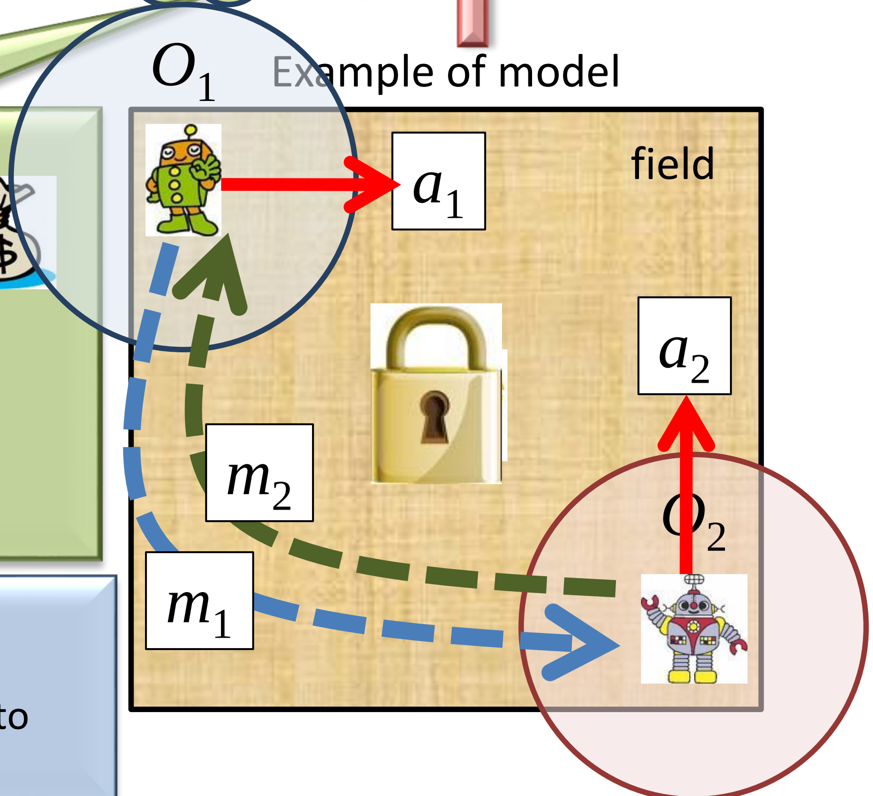
Formulation

- R : a reward function
- e.g., the treasure obtained by agents

- T : a time horizon

Example of model



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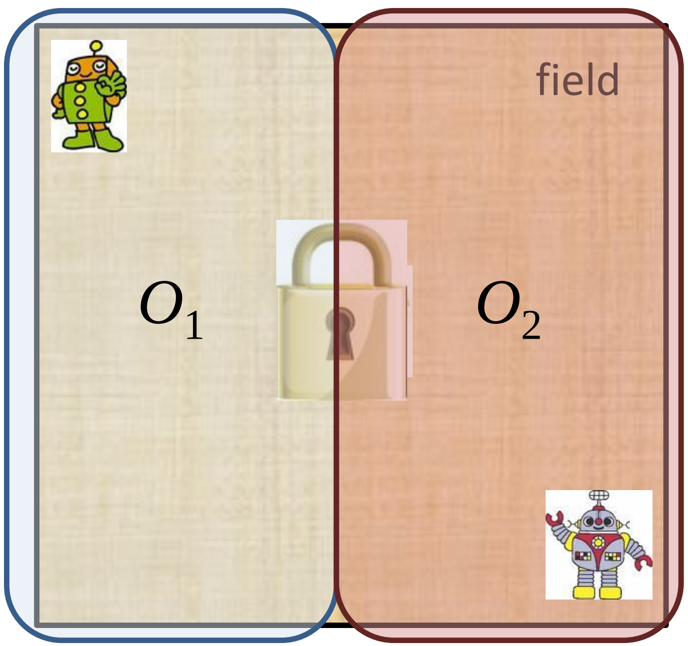
Jointly Fully Observable Dec-POMDP-Com

[Goldman+ 04]

- The Dec-POMDP-Com such that the combination of the agents' observations leads to the global state.

Jointly fully Observable Dec-POMDP-Com



$o_1 + o_2 =$ global state
(That is Jointly fully observable)

Deterministic Dec-POMDP-Com

- The model where P and O on the definition are constrained.
- Dec-POMDP-Com := $< I, S, \boldsymbol{\Omega}, \boldsymbol{A}, \boldsymbol{M}, C, P, O, R, T >$

Deterministic Dec-POMDP-Com

- The model where P and O on the definition are constrained.
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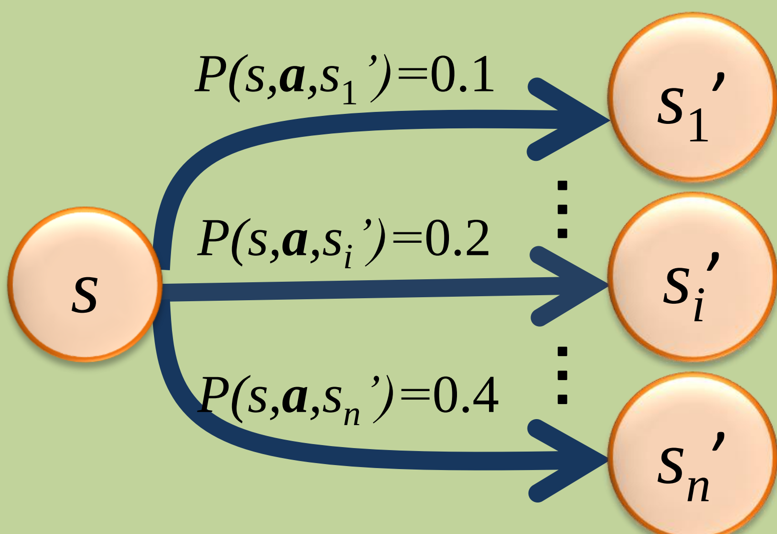
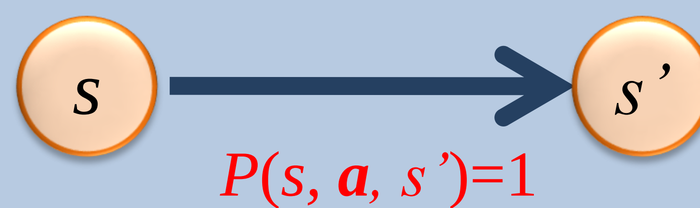
Restriction 1 : Deterministic transitions

- P is a transition probability function



$n = |S|$

- For any state $s \in S$ and any joint action $\boldsymbol{a} \in \boldsymbol{A}$, there exists a state $s' \in S$ such that $P(s, \boldsymbol{a}, s') = 1$.



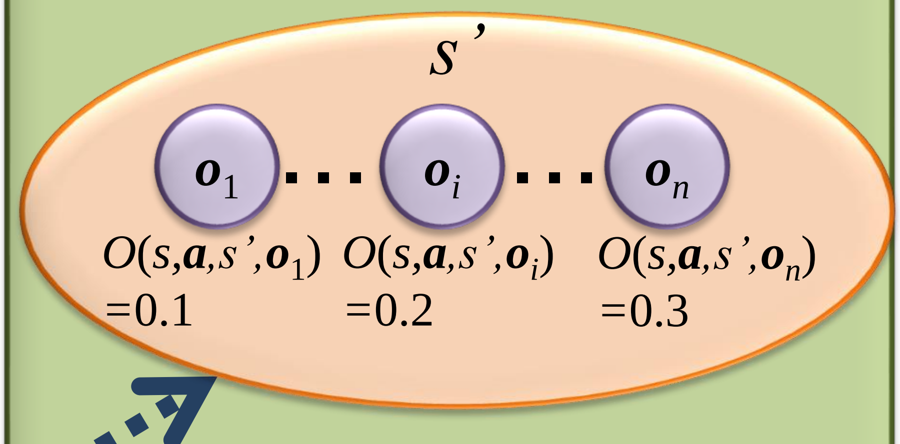
The next global state is decided uniquely.

Deterministic Dec-POMDP-Com

- The model where P and O on the definition are constrained.
- Dec-POMDP-Com := $< I, S, \boldsymbol{\Omega}, \boldsymbol{A}, \boldsymbol{M}, C, P, O, R, T >$

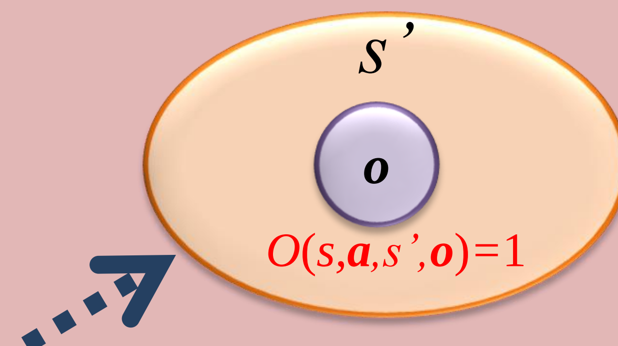
Restriction 2 : Deterministic observable

- O is a observation probability function



$n = |\boldsymbol{\Omega}|$

- For any state $s, s' \in S$ and any joint action $\boldsymbol{a} \in \boldsymbol{A}$, there exists a joint observation $\boldsymbol{o} \in \boldsymbol{\Omega}$ such that $O(s, \boldsymbol{a}, s', \boldsymbol{o}) = 1$.



The current observation is decided uniquely.

Deterministic Dec-POMDP-Com

- The model where P and O on the definition are constrained.
- Dec-POMDP-Com := $< I, S, \boldsymbol{\Omega}, \boldsymbol{A}, \boldsymbol{M}, C, P, O, R, T >$

Restriction 1 : Deterministic transitions	The next global state is decided uniquely.
Restriction 2 : Deterministic observable	The current observation is decided uniquely.

- When Dec-POMDP-Com has Restriction 1 and 2, it is called Deterministic Dec-POMDP-Com

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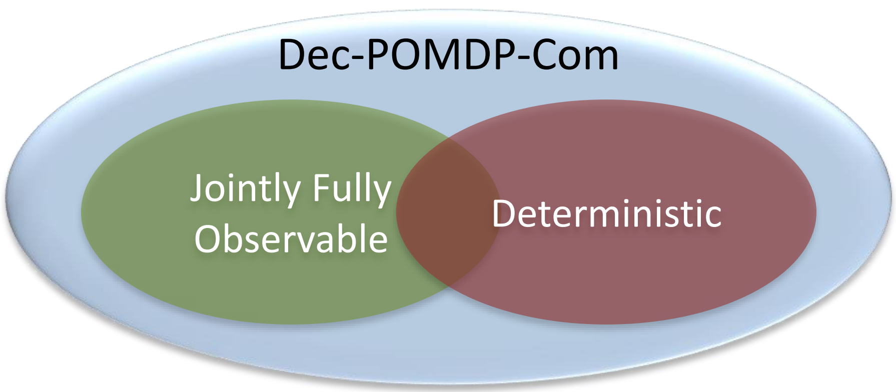
Main results

- Corollary 1 :
  Minimum required sizes $|M_i|$ for Signal Learning on *Jointly Fully Observable Dec-POMDP-Com*

- Theorem 2 :
  Minimum required sizes $|M_i|$ for Signal Learning with Messages on *Deterministic Dec-POMDP-Com*



Theorem 1 [Goldman 04]

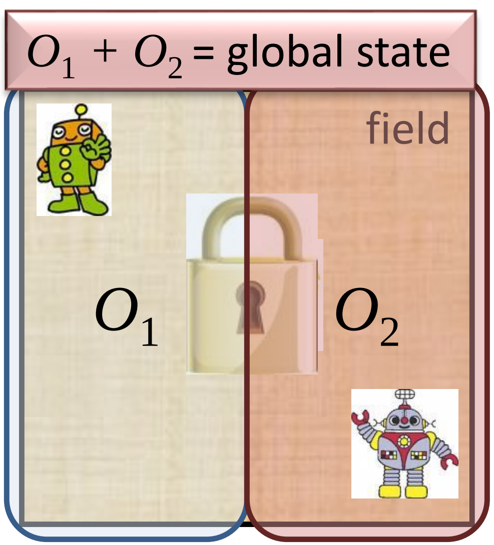
Theorem 1

For any jointly fully observable Dec-POMDP-Com, the following equation holds.

$$\forall \boldsymbol{M},\ \max_{\boldsymbol{\delta} \in \boldsymbol{D}} V^{T}_{\boldsymbol{\delta},\boldsymbol{M}}(s_0) \leq \max_{\boldsymbol{\delta} \in \boldsymbol{D}} V^{T}_{\boldsymbol{\delta},\boldsymbol{M}'}(s_0)$$

*the value of the optimal joint policy with respect to any joint message set **M***

*the value of the optimal joint policy with respect to the joint message set **M' := Ω**.*



This theorem means that the optimal communication policy of each agent is to send its own observation in jointly fully observable Dec-POMDP-Com (i.e. for agent i, $m_i := o_i$).

Each agent can always know the current global state by own observation and received message.
(e.g., for agent 1, $o_1 + m_2 = o_1 + o_2$ = global state)
Therefore, each agent always perform the optimal actions.

Corollary 1

Corollary 1

For any jointly fully observable Dec-POMDP-Com, if the size $|M_i|$ of the message set of each agent i satisfies the condition,

$$|M_i| \geqq |\Omega_i|$$

then the following equation holds:

$$\max_{\delta \in D^{SL}} V_{\delta}^{T}(s_0) = \max_{\delta' \in D} V_{\delta'}^{T}(s_0)$$

the value of the optimal joint policy on SL

the value of the optimal joint policy with history

From theorem 1 by Goldman,
the optimal communication policy of each agent is
to send its own observation in jointly fully observable Dec-POMDP-Com.

Therefore, If each agent has $|M_i|$ that is larger than $|\Omega_i|$,
it is possible to constructing the optimal policy such that each agent can send its own observation.

Theorem 2

Theorem 2

For any deterministic Dec-POMDP-Com, if the size $|M_i|$ of the message set of each agent i satisfies the condition,

$$|M_i| \geqq \max_{j \in I} \max_{o \in \Omega_j} |S_j^{obs}(o)|$$

then the following equation holds:

$$\max_{\boldsymbol{\delta} \in \boldsymbol{D}^{SLM}} V_{\boldsymbol{\delta}}^T(s_0) = \max_{\boldsymbol{\delta}' \in \boldsymbol{D}} V_{\boldsymbol{\delta}'}^T(s_0)$$

the value of the optimal joint policy on SLM

the value of the optimal joint policy with history

First, I explain a function S_j^{obs}.

S_j^{obs}

- Deterministic Dec-POMDP-Com has the following properties.

Deterministic transitions ⇒ The next global state is decided uniquely.

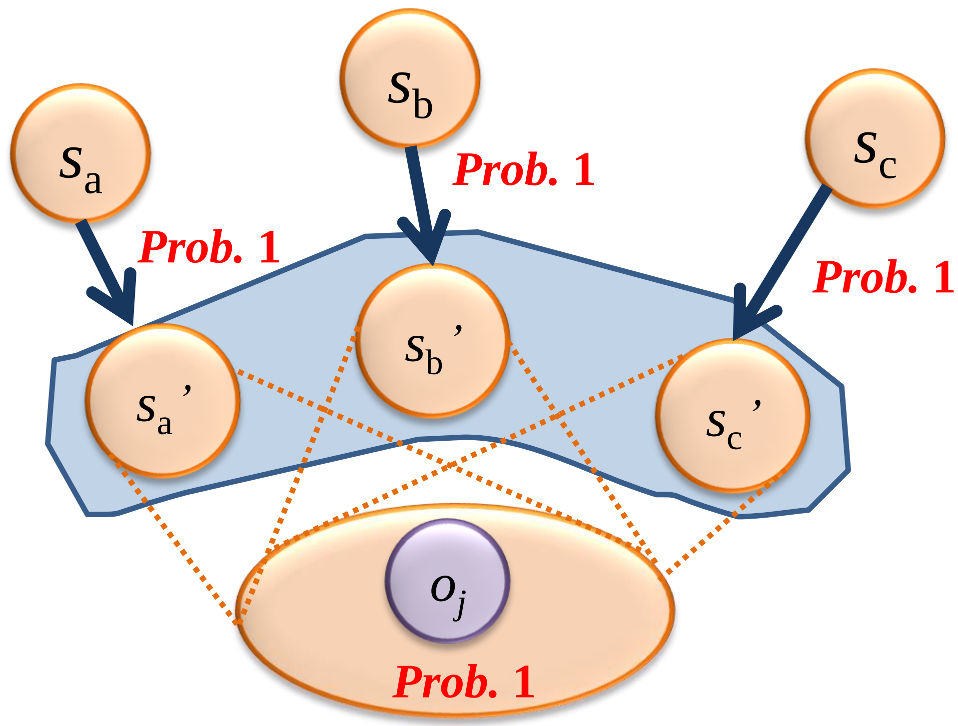
Deterministic observable ⇒ The observation is decided uniquely.

There exists some transitions such that agent *j* observes the same observation.

From the properties on deterministic Dec-POMDP-Com, we can compute the following function.

$$S_j^{obs}(o_j) = \{s_a', s_b', s_c'\}$$

S_j^{obs} returns the set of all states where agent *j* observes o_j.



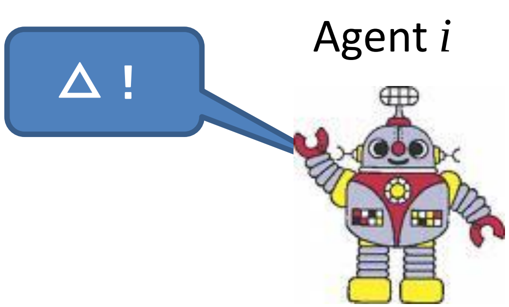
Proof sketch of Theorem2

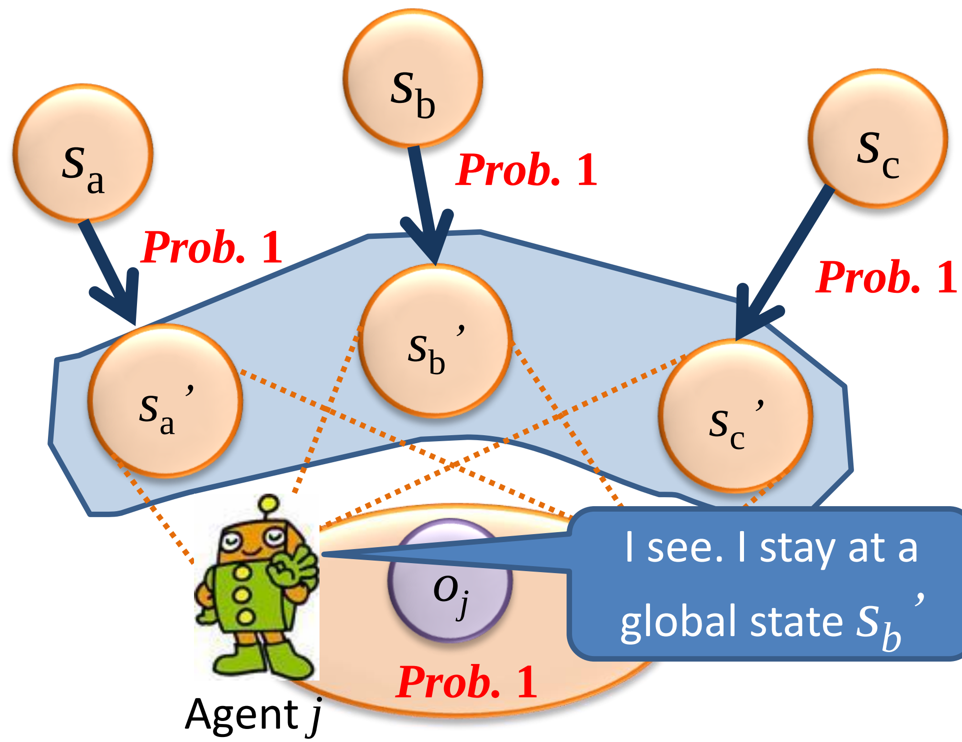
The condition of theorem 2 is $|M_i| \geqq \max_{j \in I} \max_{o \in \Omega_j} |S_j^{obs}(o)|$

The condition shows that
agent j can know the global state based on the message received from agent i by setting the set of message which have the maximum size of $S_j^{obs}(o_j)$.

$S_j^{obs}(o_j) = \{s_a', s_b', s_c'\}$

$M_i = \{□, △, ○\}$





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Conclusion

- We defined deterministic Dec-POMDP-Com for theoretical analysis

Restriction 1 : Deterministic transitions	The next global state s' is decided uniquely.
Restriction 2 : Deterministic observable	The observation $\boldsymbol{o}$ is decided uniquely.

- We showed *Minimum required sizes* $|M_i|$ for
  - Signal Learning on *Jointly Fully Observable* *Dec-POMDP-Com* at corollary 1
  - Signal Learning with Messages on *Deterministic* *Dec-POMDP-Com* at Theorem 2